Exercise session 1

Definition 1. Let f, g be functions $\mathbb{N} \to \mathbb{R}$.

$$f \in \mathcal{O}(g) \quad \text{means} \quad (\exists c, n_0 \in \mathbb{N})(\forall n \geq n_0)(|f(n)| \leq c|g(n)|) \qquad (\leq)$$

$$f \in o(g) \quad \text{means} \quad (\forall \varepsilon > 0)(\exists n_0 \in \mathbb{N})(\forall n \geq n_0)(|f(n)| < \varepsilon|g(n)|) \qquad (<)$$

$$\text{or equivalently} \quad \lim_{n \to \infty} \frac{|f(n)|}{|g(n)|} = 0$$

$$f \in \Omega(g) \quad \text{means} \quad g \in \mathcal{O}(f) \qquad (\geq)$$

$$f \in \omega(g) \quad \text{means} \quad g \in o(f) \qquad (>)$$

$$f \in \Theta(g) \quad \text{means} \quad f \in \mathcal{O}(g) \cap \Omega(g) \qquad (=)$$

Exercise 1.

1. Let $f(n) = pn^3 + qn^2 + rn + s$ for some $p, q, r, s \in \mathbb{R}$.
   Show $f(n) \in \mathcal{O}(n^3)$ and $f(n) \in o(n^4)$.

2. Show $\sin(n) \in \mathcal{O}(1)$ and $\sin(n) \notin o(1)$.

3. Show that $\mathcal{O}(f+g) = \mathcal{O}(\max(f,g))$ for non-negative functions f, g (i.e. $f(n) \geq 0$ and $g(n) \geq 0$ for all n), where $f + g$ is the function $x \mapsto f(x) + g(x)$ and $\max(f, g)$ is the function $x \mapsto \max(f(x), g(x))$.

4. Show $n^{\log n} \in \mathcal{O}(2^n)$.