Complexity Theory

Homework Sheet 4

(Turn in before the exercise session of Thursday 1 Oct.)

24 September 2015

Exercise 1. In this exercise, all graphs are directed graphs. We define the following decision problem:

$$A = \{\langle G, s, t\rangle \mid \text{The graph } G \text{ contains a vertex } v, \text{ not reachable from vertex } s, \text{ such that } t \text{ is reachable from } v.\}$$

Show that this problem is contained in NL.

Exercise 2. Define the complexity class

$$\mathrm{DP} = \{A \cap B \mid A \in \mathrm{NP}, B \in \mathrm{coNP}\}.$$

We say an undirected graph G has a *clique* of size k if there exists a subset S of k vertices such that all pairs of vertices in S have an edge between them.

$$\mathrm{ECLIQUE} = \{\langle G, k\rangle \mid \text{the largest clique in the graph } G \text{ has exactly } k \text{ vertices}\}.$$

(a) Show that ECLIQUE $\in \Sigma_2^p \cap \Pi_2^p$.

(b) Show that ECLIQUE $\in$ DP.

(c) Show that if DP $\subseteq$ NP, then the polynomial-time hierarchy collapses.

Exercise 3. Define P/*log* as the class of sets A for which there is an advice function $\alpha : \mathbb{N} \to \{0,1\}^{O(\log n)}$ and a polytime machine M such that for any string x of length *at most* n, $x \in A \iff M(x, \alpha(n)) = 1$.

Show that if SAT is in P/*log*, then P = NP.

Exercise 4. For a set $S \subseteq \{0,1\}^*$, let $S^{\leq n}$ be the strings in S of length at most n. Such a set S is called *sparse* if there is a polynomial p such that the size of $S^{\leq n}$ is at most $p(n)$ for all n. Prove that P/*poly* $= \{A \mid$ there is a sparse set S such that $A \in \mathrm{P}^S\}$.