Complexity Theory

Homework Sheet 3

Turn in before the exercise session on Thursday 24 Sep.

17 September, 2015

Exercise 1. Is there an oracle such that, relative to this oracle, ...? If so, then exhibit such an oracle and prove it works. If not, prove why not.

(a) $\mathrm{P} = \mathrm{EXP}$

(b) $\mathrm{coNP} \subseteq \mathrm{P}$ and $\mathrm{NP} \not\subseteq \mathrm{P}$

(c) $\mathrm{DTIME}(n) = \mathrm{DTIME}(n^2)$

(d) $\mathrm{NP} = \mathrm{coNP} \neq \mathrm{EXP}$

For example, in **(a)** you have to either show: there exists an oracle A such that $\mathrm{P}^A = \mathrm{EXP}^A$, or: such an oracle does not exist. In **(b)** you show either: there exists an oracle A such that $\mathrm{coNP}^A \subseteq \mathrm{P}^A$ and $\mathrm{NP}^A \not\subseteq \mathrm{P}^A$, or: such an oracle does not exist.

Exercise 2. Let C be the class of sets decidable by Turing machines which use polynomial space, but are not allowed to reuse space. I.e., the Turing machine can write over blank squares, but it can neither erase nor overwrite previously used squares – it can only read them.

(a) Prove that $\mathrm{C} \subseteq \mathrm{P}$.

(b) Prove that $\mathrm{P} \subseteq \mathrm{C}$.

Definition 1. **(I)** We say that a set A is *1-query length-decreasing self-reducible* if there is a polytime oracle Turing machine M, such that

$$x \in A \iff M^A(x) = 1$$

and the computation of $M^A(x)$ only queries a single string of length strictly less than $|x|$.

(II) We say that a set A is *length-decreasing self-reducible* if there is a polytime oracle Turing maching M, such that

$$x \in A \iff M^A(x) = 1,$$

and the computation of $M^A(x)$ only queries A on strings of length strictly less than $|x|$.

(III) We say that a set A is *lexicographically self-reducible* if there is a polytime oracle Turing machine M, such that

$$x \in A \iff M^A(x) = 1$$

and the computation of $M^A(x)$ only queries A on strings y for which either $|y| < |x|$, or ($|y| = |x|$ and y is lexicographically smaller than x).

Exercise 3. **(a)** Show that every 1-query length-decreasing self-reducible set is in P.

(b) Show that every length-decreasing self-reducible set is in PSPACE.

(c) Show that every lexicographically self-reducible set is in EXP.

Hint: How many strings y are there such that $|y| < |x|$ or ($|y| = |x|$ and y is lexicographically smaller than x)?