Complexity Theory

Homework Sheet 3

(Turn in before the lecure of Wednesday 23 Apr.)

14 April, 2014

Exercise 1. Is there an oracle such that ...? If so, then exhibit such an oracle and prove it works. If not, prove why not.

(a) P = EXP

(b) coNP $\subseteq$ P and NP $\nsubseteq$ P

(c) NP $\neq$ EXP

Exercise 2. Let C be the class of sets decidable by Turing machines which use polynomial space, but are not allowed to reuse space. I.e., the Turing machine can write over blank squares, but it can neither erase nor overwrite previously used squares – it can only read them.

(a) Prove that C $\subseteq$ P.

(b) Prove that P $\subseteq$ C.

Definition 1. **(I)** We say that a set A is *1-query length-decreasing self-reducible* if there is a polytime oracle Turing machine M, such that

$$x \in A \iff M^A(x) = 1$$

and the computation of $M^A(x)$ only queries a single string of length strictly less than $|x|$.

(II) Let $\oplus$ denote the exclusive disjunction. We say that a set A is *2-query parity length-decreasing self-reducible* if there are two polytime-computable length-decreasing functions f_1, f_2 and there is some constant ℓ such that for all x of length at least ℓ we have

$$x \in A \iff (f_1(x) \in A) \oplus (f_2(x) \in A)$$

(III) We say that a set A is *length-decreasing self-reducible* if there is a polytime oracle Turing maching M, such that

$$x \in A \iff M^A(x) = 1,$$

and the computation of $M^A(x)$ only queries A on strings of length strictly less than $|x|$.

Definition 2. We say that a language L is in the complexity class $\oplus$P (pronounced as 'parity P') if there exists a polytime Turing machine M and a polynomial p such that

$$x \in L \iff \text{there exist an odd number of } u \in \{0,1\}^{p(|x|)} \text{ s.t. } M(x,u) = 1$$

Exercise 3. (a) Show that every 1-query length-decreasing self-reducible set is in P.

(b) Show that every 2-query parity length-decreasing self-reducible set is in $\oplus$P.

Hint: view the queries as a tree, with x at the root, and let the witness u be a path through this tree.

(c) Show that every length-decreasing self-reducible set is in PSPACE.